A Proposal of Low-Loss Leaky Waveguide for Submillimeter Waves Transmission

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***Abstract*—A low-loss dielectric leaky waveguide which carries most of the power in air is proposed for long-distance transmission of submillimeter waves, and its guiding mechanism is fully analyzed by using a two-dimensional slab-waveguide model. It is shown that transmission losses of the guide can become several decibels per kilometer and are not significantly affected by dielectric losses of material used for practical guide parameters.**

I. Introduction

ONE OF THE most difficult problems in long-distance transmission of submillimeter waves is lack of waveguide with low losses: The energy of waves scatters and attenuates in dielectrics or metals of guiding media present. One of the ways to overcome the high dielectric or metallic losses is to guide the wave in low-loss regions, e.g., in air, as far as possible.

A tube waveguide or *O*-guide seems to be a possible candidate for such waveguides [1]–[3]. As far as the guided or bound mode whose energy is trapped in the higher index region is transmitted, still one cannot expect low transmission losses without sacrificing additional losses due to bends. However, if one considers leaky modes, the situation met in ordinary surface waveguides is completely overcome.

In this paper, we propose a leaky waveguide with extremely low losses and show that it is possible for the energy of the leaky modes to be tightly confined in the region of air at properly chosen guide parameters, and that the attenuation constant becomes extremely small, not only in submillimeter wavelengths but also in optical wavelengths.

II. Derivation of Characteristic Equation

First, we will analyze a dielectric tube waveguide as one of the possible leaky waveguides, whose refractive-index profile is shown in Fig. 1, where n_0 represents the refractive index of air, $a>1$, and a two-dimensional slab-waveguide uniform in y and z direction is assumed. We will seek a mode whose energy concentrates in the region of air between two parallel dielectric sheets, i.e., $|x|<T$. The electric field E_y of the leaky TE_n^m mode of the waveguide is expressed by

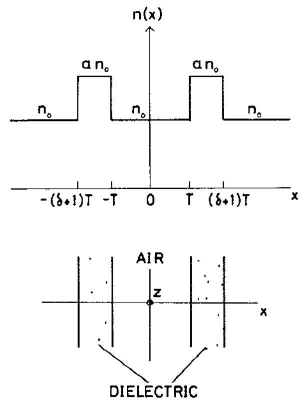

Fig. 1. Refractive-index profile of the tube waveguide.

$$E_y=\begin{cases}\cos\left(u\dfrac{x}{T}-\dfrac{n\pi}{2}\right), & 0<x<T\\[2mm] \dfrac{\cos(u-n\pi/2)}{\cos(u_1+\varphi)}\cos\left(u_1\dfrac{x}{T}+\varphi\right), & T<x<(1+\delta)T\\[2mm] \dfrac{\cos(u-n\pi/2)}{\cos(u_1+\varphi)}\cos\left[(\delta+1)u_1+\varphi\right]e^{-ju[x-(1+\delta)T]/T}, & x>(1+\delta)T\end{cases}\tag{1}$$

where time and z dependences of $\exp j(\omega t-\beta z)$ are suppressed. The transverse complex phase constant u/T and u_1/T are defined by

$$u^2=(n_0^2k_0^2-\beta^2)T^2\tag{2}$$

$$u_1^2=(a^2n_0^2k_0^2-\beta^2)T^2.\tag{3}$$

Following the standard steps of solving a boundary-value problem, we obtain simultaneous transcendental equations to determine u and u_1 as follows:

$$\tan\left(u-\frac{n\pi}{2}\right)=j\frac{u_1}{u}\cdot\frac{u+ju_1\tan(\delta u_1)}{u_1+ju\tan(\delta u_1)}\tag{4}$$

$$u_1^2-u^2=(a^2-1)(n_0k_0T)^2\equiv v^2\tag{5}$$

where the condition $\mathrm{Re}(u)>0$ must be imposed. Once u is determined, β is obtained from (2). Note that leaky modes appear below the cutoff frequency v_c of $TE^{(m)}$ guided

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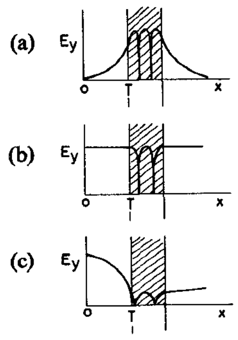

Fig. 2. Schematic views of the field distributions for (a) $v>v_c$, (b) $v=v_c$, and (c) $v<v_c$.

mode, whose energy concentrates at or near the region of dielectric. We use notations $TE^{(m)}$ and TE_n^m to distinguish the guided mode with leaky mode, respectively, and n and m describe the number of peaks of electric-field intensity in $0<x<T$ and $T<x<(1+\delta)T$, respectively.

Putting $u=0$ in (4) and (5), we can determine the cutoff frequency by

$$v_c \tan(\delta v_c)=\begin{cases} 0 & \text{(even modes)} \\ 1 & \text{(odd modes).}\end{cases} \tag{6}$$

When m increases, v_c becomes

$$v_c=\frac{\pi}{2\delta}\times\begin{cases} m & \text{(even modes)} \\ m-1 & \text{(odd modes).}\end{cases} \tag{7}$$

Fig. 2 shows schematic view of the field distributions for $v>v_c$ (guided mode propagation), $v=v_c$ (cutoff condition), and $v<v_c$ (leaky mode propagation).

III. Numerical Results

Typical examples of the attenuation constants α of leaky modes obtained numerically are shown in Figs. 3 and 4 for $a=1.5$ and $\delta=0.5$ as a function of the normalized frequency v. The number m on each curve is the order associated with the guided $TE^{(m)}$ mode. It is clear that a leaky mode associated with the guided $TE^{(0)}$ mode never exists because it has no cutoff frequency different from zero. Also, we cannot numerically find a leaky mode associated with the guided $TE^{(1)}$ mode whose cutoff frequency is 1.30654 for $\delta=0.5$. The curves of the attenuation constant decrease oscillatorily with the normalized frequency, increasing and decreasing rapidly near the cutoff frequency of the guided mode.

Using the transverse transmission-line model [4], we can well approximate the attenuation constant of the leaky TE_n^m mode by

$$\alpha=n_0k_0(a^2-1)\frac{u_\infty^3}{v^2\sin^2(\delta v)+4u_\infty^2}\cdot\frac{1}{v^2} \tag{8}$$

where u_∞ is $(n+1)\pi/2$. Equation (8) shows that the minimum attenuation constant $\alpha_{\min}$ is attained locally at

$$v=\left(l+\frac{1}{2}\right)\pi/\delta \tag{9}$$

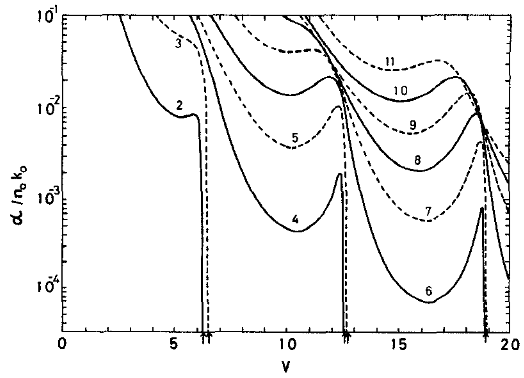

Fig. 3. Attenuation constant of leaky modes versus normalized frequency v in the case of $a=1.5$ and $\delta=0.5$. Numbers m on each curve correspond to the orders of associated guided $TE^{(m)}$ modes. Arrows depict the cutoff frequencies.

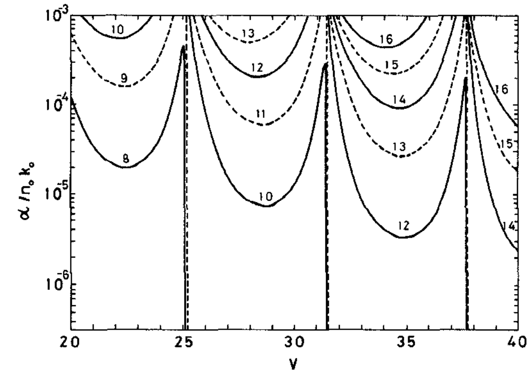

Fig. 4. Attenuation constant of leaky modes versus normalized frequency v in the case of $a=1.5$ and $\delta=0.5$. Numbers m on each curve correspond to the orders of associated guided $TE^{(m)}$ modes.

and $\alpha_{\min}$ is expressed by

$$\alpha_{\min}=n_0k_0(a^2-1)\frac{u_\infty^3}{v^2+4u_\infty^2}\cdot\frac{1}{v^2} \tag{10}$$

where l stands for

$$l=(m-n-2)/2. \tag{11}$$

When the normalized frequency v is large, $\alpha_{\min}$ is approximated by

$$\alpha_{\min}=n_0k_0(a^2-1)\frac{u_\infty^3}{v^4} \tag{12}$$

or

$$\alpha_{\min}=\frac{n_0k_0}{a^2-1}\cdot\frac{u_\infty^3}{(n_0k_0T)^4}. \tag{13}$$

Fig. 5 shows the attenuation constant $\alpha_{\min}$ of the TE_0^m mode as a function of the guide half-width T for several wavelengths, where $n_0=1$ and $a=1.5$ are assumed. The mode number m is chosen as the maximum number when (9) is satisfied. One can see that a practical long-distance waveguide can be realized not only in submillimeter wavelengths but also in infrared wavelengths with moderate guide parameters. One should also notice that the attenuation constants of the higher order modes increase with

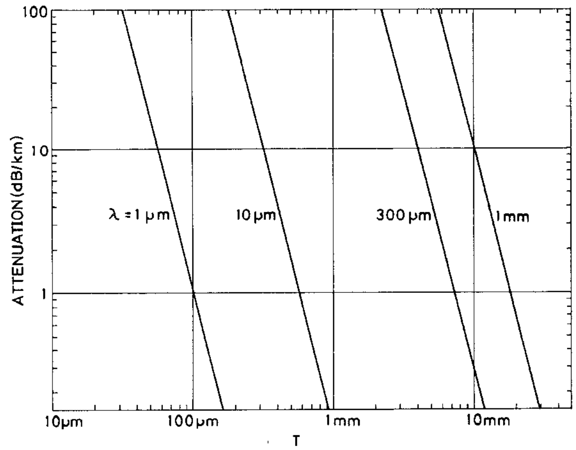

Fig. 5. Attenuation constant of the leaky $TE_0^{(m)}$ mode as a function of the guide half-width T for various wavelengths when (10) is satisfied, where $a=1.5$ and $n_0=1$ are assumed.

proportion to $(n+1)^3$. For example, the transmission loss of the leaky TE_1^m mode is larger than that of the TE_0^m mode by eight times, which means that higher order leaky modes can be damped out by introducing a reasonable transmission loss to the dominant TE_0^m mode and practical single-mode transmission can be realized in spite of the large-core diameter.

At the same time, we have to mention a point about surface-wave (bound) modes which are also present along this structure. By noticing that the energy of the surface-wave mode is still trapped in the dielectric material, the transmission loss of this mode in the case of high material loss present can only be reduced when the fields spread out in air region, i.e., $|x|>(1+\delta)T$. However, this will significantly increase the bending losses of the bound modes.

In the case of the dielectric hollow waveguide [5] which can also support leaky modes, the attenuation constant of the TE_n mode is given by [6]

$$\alpha=\frac{n_0k_0}{(a^2-1)^{\frac{1}{2}}}\cdot\frac{u_\infty^2}{(n_0k_0T)^3} \tag{14}$$

which is larger than that of the tube waveguide by several orders of magnitude, and the mode suppression of higher order modes is worse than that of the tube waveguide proposed here.

In order to understand the low-loss mechanisms of the leaky modes in the tube waveguide, field distributions of the leaky TE_0^m modes are shown in Fig. 6 at the normalized frequencies where the attenuation constant takes a local minimum for $\delta=0.5$. Most of the power of the leaky mode is shown to be trapped in the inner region of air, which means that the attenuation constant of the leaky mode is not significantly affected even if the dielectric material separating two regions of air has some loss. In fact, by letting α_d be the plane wave attenuation constant in the dielectric, the attenuation constant $\alpha_{\min}$ is calculated by using a perturbation theory as

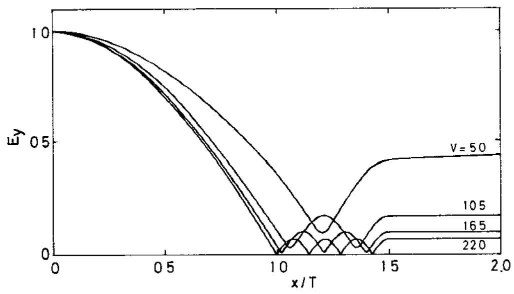

Fig. 6. Field distributions of the leaky $TE_0^{(m)}$ $(m=2,4,6,8)$ modes, where the attenuation constants take local minima.

$$\alpha_{\min}=\frac{n_0k_0}{a^2-1}\cdot\frac{u_\infty^3}{(n_0k_0T)^4}\left[1+\frac{an_0k_0T}{u_\infty}(\delta\alpha_dT)\right] \tag{15}$$

when (10) is satisfied. It is clear that the second term of (15) can be made small by choosing small δ even if α_d is large. Typically, for the leaky waveguide with the core radius $T=4$ mm operated at $\lambda=300$ μm, the transmission loss of the TE_0^m mode is around 10 dB/km in the case of $n_0=1$ and $a=1.5$. Even if we assume the material loss of $2\alpha_d=1$ dB/m, the increment of the additional loss is only 2 percent for $\delta=0.5$. For the infrared wavelengths, more larger values of δ are permitted in keeping the additional loss due to material loss small.

Equations (9) and (12) show that the attenuation constant of the leaky modes does not change if $(l+1/2)/\delta$ is constant. However, frequency characteristics of the attenuation constant do change significantly with δ. Let v_0 be the normalized frequency characterized by (9). If the condition

$$|v-v_0|<\frac{\pi}{4\delta} \tag{16}$$

is satisfied, the attenuation constant does not increase up to twice that given by (12). From this point of view, the leaky waveguide with small δ will be preferable to design the guide with broad frequency characteristics and also the wall deformation may not seriously affect on the loss characteristics.

For the TM_n^m mode, the attenuation constant corresponding to (15) is expressed as follows:

$$\alpha_{\min}=\frac{n_0k_0}{a^2-1}\cdot\frac{u_\infty^3}{(n_0k_0T)^4}\left[a^4+\frac{a^3n_0k_0T}{u_\infty}(\delta\alpha_dT)\right]. \tag{17}$$

So far, we have analyzed the characteristics of the low-loss waveguide by using a two-dimensional slab waveguide model. For the tube waveguide with a round cross section, we can show that (8)–(17) so far obtained, are also valid for the TE_{0n} and TM_{0n} modes if T is taken as the radius of the core and u_∞ is taken as the nth root of $J_1(x)=0$ except zero [4].

Finally, we emphasize that for low-loss transmission, the refractive index of the inner medium need not be higher than that of the surrounding infinite medium [7], [8] and that the structure of the waveguide is quite simpler than those in [9]–[11].

IV. Conclusion

A leaky waveguide with extremely low losses is proposed for long-distance transmission of submillimeter waves, and its guiding mechanism is fully analyzed by using a two-dimensional slab waveguide model. It is also shown that long-distance transmission is possible in submillimeter through optical wavelengths. Attenuation constants of the leaky TE_{0n} and TM_{0n} modes in round structure are also presented.

The excitation efficiency of the leaky modes and additional losses due to bends will be analyzed in future publications.

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References

[1] M. Sugi and T. Nakahara, "*O*-guide and *X*-guide: An advanced surface wave transmission concept," *IRE Trans. Microwave Theory Tech.*, vol. MTT-7, no. 3, pp. 366–369, July 1959.

[2] M. M. Z. Kharadly and J. E. Lewis, "Properties of dielectric tube waveguides," *Proc. Inst. Elec. Eng.*, vol. 116, no. 2, pp. 214–224, 1969.

[3] M. Miyagi and S. Nishida, "Transmission characteristics of a dielectric tube waveguide with an outer higher-index cladding," *Electron Lett.*, vol. 13, no. 10, pp. 274–275, May 1977.

[4] ——, "A low-loss leaky waveguide," presented at the meeting of P. G. on Microwaves, IECE of Japan, 1979; Paper MW79-67, (in Japanese).

[5] E. A. J. Marcatili and R. A. Schmeltzer, "Hollow metallic and dielectric waveguides for long distance optical transmission and lasers," *Bell Syst. Tech. J.*, vol. 43, pp. 1783–1809, July 1964.

[6] D. Marcuse, *Theory of Dielectric Optical Waveguides*. New York: Academic, pp. 43–49, 1974.

[7] E. A. J. Marcatili, "Light transmission in a multiple dielectric (gaseous and solid) guide," *Bell Syst. Tech. J.*, vol. 45, no. 1, pp. 97–103, Jan. 1966.

[8] K. Yamamoto, "A low-loss dielectric waveguide for millimeter and submillimeter wavelengths," *Trans. IECE of Japan*, vol. J61-B, no. 7, pp. 616–623, July 1978 (in Japanese).

[9] R. P. Larsen and A. A. Oliner, "A new class of low-loss reactive-wall waveguides," in *IEEE-MTT Int. Microwave Symp. Dig.*, (Boston, MA), May 8–11, 1967.

[10] A. A. Oliner, "A new class of reactive-wall waveguides for low-loss applications," in *Progress in Radio Science 1966–1969*, vol. 3, W. V. Tilston and M. Sauzade, Eds. URSI: Brussels, 1971, pp. 19–35.

[11] P. Yeh and A. Yariv, "Theory of Bragg fiber," *J. Opt. Soc. Amer.*, vol. 68, no. 9, pp. 1196–1201, Sept. 1978.

A Multilayer Fiber Guide with Rectangular Core

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***Abstract*—An approximate analysis is given for multilayer dielectric waveguides with rectangular core. Dispersion curves are calculated for several modes over a range of parameters. Some comments related to the design of such guides for use with planar structures such as semiconductor lasers are given.**

Fiber guides with rectangular cores are important for compatibility with many integrated optics devices to which they may be connected [1]–[3]. Although exact analysis of the rectangular geometry has not been done, the available approximate analyses [4], [5], [14] are useful, provided there is a single cladding material. There may, however, be advantage in using two or more layers with different indices, as shown in Fig. 1. Planar and circular cylindrical versions of the multilayer dielectric guides have shown advantages in propagation

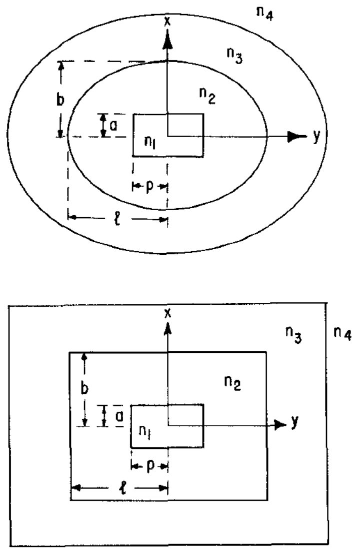

Fig. 1. (a) Fiber guide in cross section. (b) Basic model for analysis.

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